Reply to "Comment on 'Periodic phase synchronization in coupled chaotic oscillators' "

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The phase difference in coupled chaotic oscillators exhibits a small noiselike fast fluctuation in long-term phase dynamics. The fast fluctuation can cause an error in the measurement of the period of the temporary phase locking state. We discuss why we should filter out the fast fluctuation on determination of periodic phase synchronization.

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Phase synchronization (PS) is a ubiquitous phenomenon appearing in coupled chaotic oscillators with slight parameter mismatch. In order to describe PS theoretically, the trajectory of each oscillator is transformed into a polar coordinate [1]. Then the trajectories of the two oscillators can be separated into angular, amplitude, and other additive parts. Here when we obtain the phase difference from the angular parts of the two oscillators, the amplitude and the other additive parts act as a small-amplitude noise to the phase difference dynamics (phase dynamics). This behavior is well described in Eq. (3) of Refs. [2,3].

Recently a new behavior of phase dynamics was observed, which is called periodic phase synchronization (PPS). This behavior can be found when a zero Lyapunov exponent becomes locally negative in a narrow region before PS, which is called a "dip." (For example, see the time series presented in Figs. 1(a) and 4(a) of Ref. [2].) In the dip, we can see that the phase dynamics of the oscillators periodically jumps by 2π [2]. The phenomenon is distinguishable from irregular 2π phase jumps. However, it is hard to detect the PPS state directly from the phase dynamics since, as mentioned in our previous paper [2], we can observe that the 2π phase jumps accompany a small noiselike fast fluctuation before the onset point of PS due to the amplitude and other additive parts of the oscillators. In order to determine the average length of the temporal phase locking (TPL) state precisely, we have introduced a criterion in Ref. [2] such that

$$\langle \dot{\phi} \rangle < \Lambda_c, \quad (1)$$

where $\langle \cdots \rangle$ is the running average and Λ_c is the cutoff value. As we have presented the time series of the phase dynamics and $\langle \dot{\phi} \rangle$ in Figs. 2(a) and 2(b) of Ref. [2], the criterion enables us to measure the TPL length accurately in a noisy circumstance. Because of the filtering of the fast fluctuation by a running average, the time series $\langle \dot{\phi} \rangle$ exhibits peaks when the phase dynamics jumps by 2π. Then, by taking an appropriate value of Λ_c, we can obtain the TPL length more accurately in a weak-coupling regime.

Pazó and Matías [4] say in their Comment, "In our computations, we have directly measured the phase of the oscillators to detect phase slip." In their figures, the minimum value of the coherence measure in the PPS region is similar to ours but the border of PPS is not clear. As we explained in our previous paper [2], the phase dynamics of two coupled chaotic oscillators is always perturbed by a noiselike fast fluctuation due to the dynamics of the amplitude and other additive parts. In order to analyze the phase slips it is essential to filter out the noiselike fluctuation. So without the filtering-out, the statistical results of the TPL length does not give a fine resolution for the PPS region.

To show the noiselike fast fluctuation, the temporal behavior of the phase difference of the two Rössler oscillators is given in Fig. 1 at the same value of the parameters as in our previous paper [2], where we observed PPS (Fig. 4 in Ref. [2]). It is clear in the figure that when the phase difference of the two oscillators jumps by 2π, there is an irregular fast fluctuation (the faint line). When we take a running average, the fluctuation becomes smooth (the black line). This means that when one does not filter it out, the fast fluctuation causes a serious error in the measurement of the period of the TPL length. When we obtain the derivative of the phase difference and take a running average to filter out the fast fluctuation, we can see clear peaks as shown in Fig. 2(b) in Ref. [2]. This is why we should take a running average to define a PPS state.

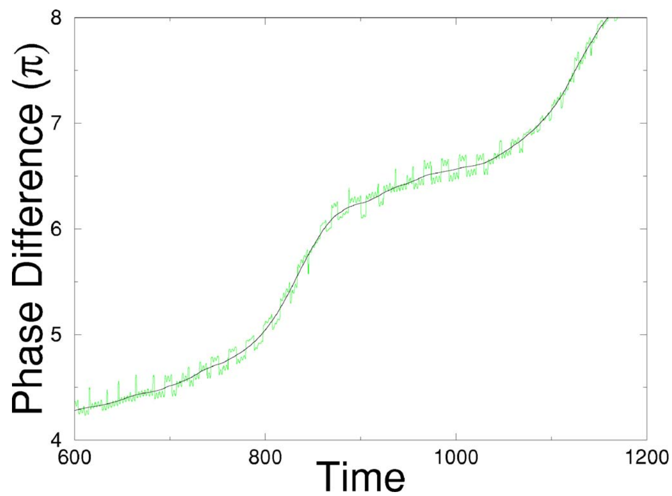

FIG. 1. (Color online) Temporal behavior of the phase difference between coupled oscillators in the region of periodic phase synchronization. The faint (green) line is the temporal behavior without filtering out the fast fluctuation and the black line with filtering.

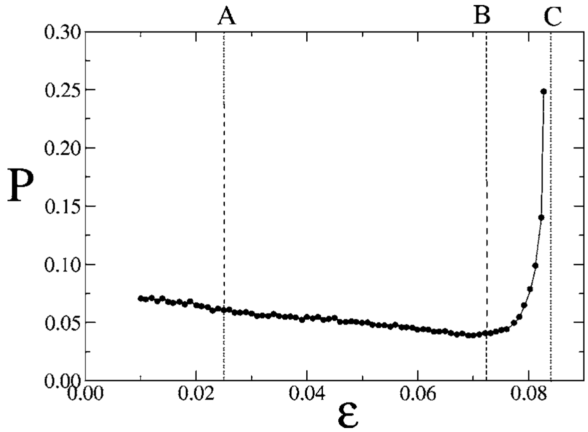

FIG. 2. Coherence measure without filtering out a noiselike fast fluctuation for the mutually coupled Rössler oscillators.

In order to define the periodicity, we introduced a coherence measure $P(\epsilon)=\sqrt{\mathrm{var}(\tau)}/\langle\tau\rangle$, where τ is the TPL length after filtering out the fast fluctuation [2]. When an error occurs in the measurement of the period of the TPL length, the error causes a serious error in $P(\epsilon)$. So if one does not filter out the fast fluctuation, it is hard to observe the dip in $P(\epsilon)$. If yes, one can see the clear dip in $P(\epsilon)$ as is shown in Figs. 3 and 4 in Ref. [2]. To show the PPS state more clearly, we obtained $P(\epsilon)$ depending on the coupling strength in the case of unidirectional coupling in Ref. [2], which is the first example.

However, when we directly measure the period of the TPL length in the PPS region without the filtering we cannot see any dip in $P(\epsilon)$, as shown in Fig. 2, in correspondence to the results of Pazó and Matías [4]. To repeat, when we do not filter out the fast fluctuation, there is a serious ambiguity in the measurement of the period. When the period of the TPL state is short, the problem is more serious. In our measurement, as we increase the running average bandwidth, the slope becomes steeper. At last, at an appropriate filtering bandwidth, we can see the dip as shown in Fig. 3 in Ref. [2].

It is well known that when the signal of a nonlinear dynamical system is periodic, one Lyapunov exponent is zero and the others negative. In the presence of noise, as the noise amplitude increases, the negative Lyapunov exponent approaches zero. In this regard, it is not surprising that PPS is closely related to the negative dip in the Lyapunov exponent. Also the dip depth in $P(\epsilon)$ depends on the negative value of the Lyapunov exponent. So the negative dip in the Lyapunov exponent is another piece of evidence of PPS. When we compare the Lyapunov exponents presented in Figs. 1 and 4 in Ref. [2], the dip in Fig. 1 is much deeper than the dip in Fig. 4. Accordingly, the minimum value of $P(\epsilon)$ in Fig. 3 is deeper than that in Fig. 4 in Ref. [2]. The authors of the Comment should have used some filtering methods to detect the PPS region accurately.

Another issue raised by Pazó and Matías [4] was that $P(\epsilon)$ converges to 1.0 as $\epsilon\to 0$ in Ref. [2], while $P(\epsilon)$ does not converge to 1.0 in their result. This discrepancy is solely caused by their measurement without the filtering of the fast fluctuation. The phenomenon of convergence to 1.0 can be easily understood when we consider the TPL region, where $\epsilon\to\epsilon_c$. Here ϵ_c is the critical value for PS. In this region, even though phase jumps are uncorrelated for large τ, $P(\epsilon)\to 1.0$. Similarly, when $\epsilon\to 0$, the coupled chaotic oscillators become independent so that the average length of the TPL is short and the phase jumps are uncorrelated like the TPL state around $\epsilon\to\epsilon_c$. These uncorrelated phase jumps cause the measure $P(\epsilon)$ to approach 1.0. To show why $P\to 1.0$ as $\epsilon\to 0$, we present the time series of $\langle\dot{\phi}\rangle$ for ϵ=0.0 in Fig. 3. Figure 3(a) is $\langle\dot{\phi}\rangle$ when we take a running average with the filtering bandwidth of 60 sec. Figure 3(b) is the time series of $\langle\dot{\phi}\rangle$ when we filter Fig. 3(a) again by taking a running average with the filtering band width of 30 sec. As is shown in the figure, the phase jumping period is very irregular because the two oscillators are highly uncorrelated. Accordingly, it is natural that $P\to 1.0$ as $\epsilon\to 0.0$.

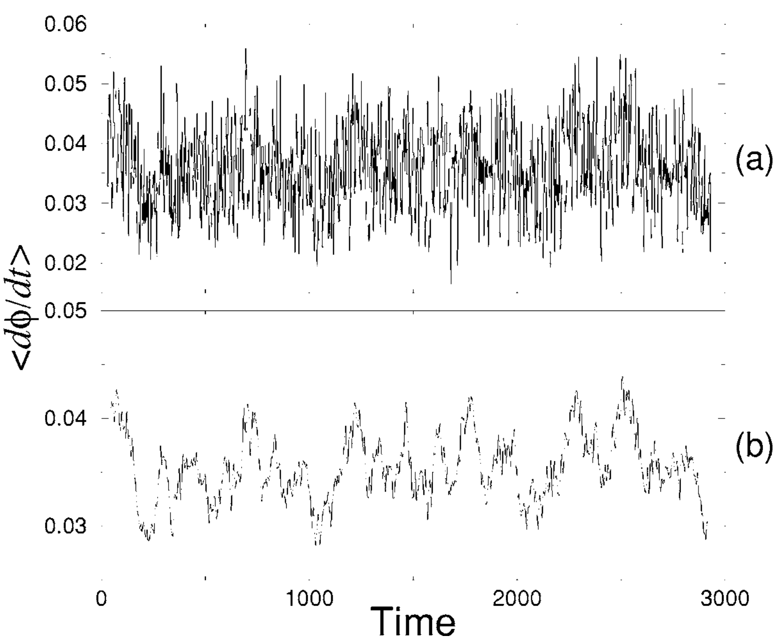

FIG. 3. Temporal behaviors of $\langle\dot{\phi}\rangle$ for ϵ=0.0. (a) is the running average with the grid size of 60 sec and (b) is the running average of (a) with the grid size of 30 sec.

Pazó and Matías [4], moreover, introduced a simplified two-dimensional model map to simulate the phenomenon [Eqs. (3) and (4) in their Comments]. It is clear that the Lyapunov exponent does not have any dip for $\epsilon < 0.1$. Consequently, we cannot see any drop to negative in $P(\epsilon)$. The negative Lyapunov exponent for $\epsilon > 0.1$ is not related to our PPS.

We also found in our previous paper a rapid increase of $P(\epsilon)$ when the zero Lyapunov exponent becomes negative for $\epsilon > 0.27$ as shown in Fig. 4 of Ref. [2]. This region is related to a TPL state with uncorrelated phase jumps. Also, in the measurement of $P(\epsilon)$ in their model map, a filtering must be used. When we do not filter out the fast fluctuation in the Rössler oscillators, there is no dip in $P(\epsilon)$; nor does $P(\epsilon)$ converge to 1.0 as $\epsilon \to 0.0$. Similarly, because they do not use a filtering in the model map, $P(\epsilon)$ does not converge to 1.0 as $\epsilon \to 0$. We emphasize here that in the model map, there is no negative dip in the Lyapunov spectrum. So there is no PPS state.

In conclusion, when we find a negative dip in the zero Lyapunov exponent before the PS region, there are periodic phase jumps. The phase jumps are also influenced by the noiselike fast fluctuation. When we filter out the fluctuation, we can see clearly the periodicity in the time series of $\langle \dot{\phi} \rangle$. The coherence measure $P(\epsilon)$ is a qualitative evidence of a periodic phase jump region. Finally, we suggest that to observe the genuine feature of PPS Pazó and Matías should have introduced an appropriate method to filter out the noiselike fast fluctuation caused by the chaotic dynamics.

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